

General Certificate of Education  
June 2006  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Monday 22 May 2006 9.00 am to 10.30 am

**For this paper you must have:**

- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

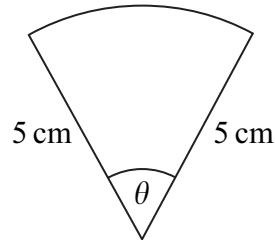
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

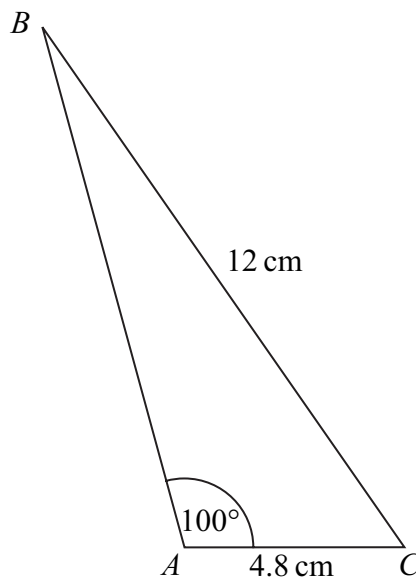
- 1 The diagram shows a sector of a circle of radius 5 cm and angle  $\theta$  radians.



The area of the sector is  $8.1 \text{ cm}^2$ .

- (a) Show that  $\theta = 0.648$ . (2 marks)
- (b) Find the perimeter of the sector. (3 marks)

- 2 The diagram shows a triangle  $ABC$ .



The lengths of  $AC$  and  $BC$  are 4.8 cm and 12 cm respectively.

The size of the angle  $BAC$  is  $100^\circ$ .

- (a) Show that angle  $ABC = 23.2^\circ$ , correct to the nearest  $0.1^\circ$ . (3 marks)
- (b) Calculate the area of triangle  $ABC$ , giving your answer in  $\text{cm}^2$  to three significant figures. (3 marks)

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. (2 marks)

(b) The sum of the first  $n$  terms of the series is 7400.

(i) Show that  $3n^2 - 2n - 7400 = 0$ . (3 marks)

(ii) Find the value of  $n$ . (2 marks)

4 (a) The expression  $(1 - 2x)^4$  can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers  $p$  and  $q$ . (3 marks)

(b) Find the coefficient of  $x$  in the expansion of  $(2 + x)^9$ . (2 marks)

(c) Find the coefficient of  $x$  in the expansion of  $(1 - 2x)^4(2 + x)^9$ . (3 marks)

5 (a) Given that

$$\log_a x = 2 \log_a 6 - \log_a 3$$

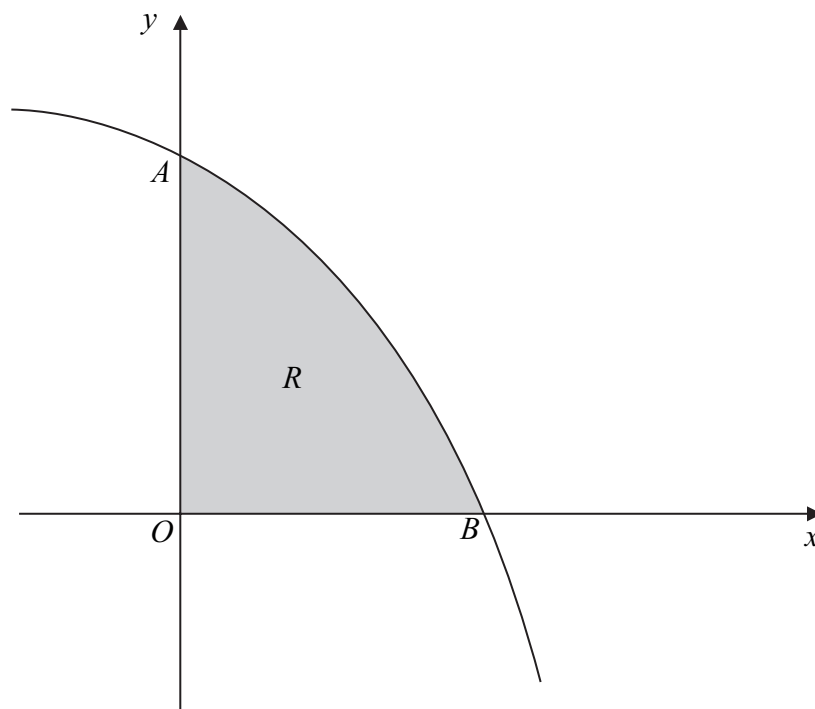
show that  $x = 12$ . (3 marks)

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express  $y$  in terms of  $a$ , giving your answer in a form not involving logarithms. (3 marks)

- 6 The diagram shows a sketch of the curve with equation  $y = 27 - 3^x$ .



The curve  $y = 27 - 3^x$  intersects the  $y$ -axis at the point  $A$  and the  $x$ -axis at the point  $B$ .

- (a) (i) Find the  $y$ -coordinate of point  $A$ . (2 marks)
- (ii) Verify that the  $x$ -coordinate of point  $B$  is 3. (1 mark)
- (b) The region,  $R$ , bounded by the curve  $y = 27 - 3^x$  and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of  $R$ . (4 marks)
- (c) (i) Use logarithms to solve the equation  $3^x = 13$ , giving your answer to four decimal places. (3 marks)
- (ii) The line  $y = k$  intersects the curve  $y = 27 - 3^x$  at the point where  $3^x = 13$ . Find the value of  $k$ . (1 mark)
- (d) (i) Describe the single geometrical transformation by which the curve with equation  $y = -3^x$  can be obtained **from** the curve  $y = 27 - 3^x$ . (2 marks)
- (ii) Sketch the curve  $y = -3^x$ . (2 marks)

7 At the point  $(x, y)$ , where  $x > 0$ , the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

- (a) (i) Verify that  $\frac{dy}{dx} = 0$  when  $x = 4$ . (1 mark)
- (ii) Write  $\frac{16}{x^2}$  in the form  $16x^k$ , where  $k$  is an integer. (1 mark)
- (iii) Find  $\frac{d^2y}{dx^2}$ . (3 marks)
- (iv) Hence determine whether the point where  $x = 4$  is a maximum or a minimum, giving a reason for your answer. (2 marks)
- (b) The point  $P(1, 8)$  lies on the curve.
- (i) Show that the gradient of the curve at the point  $P$  is 12. (1 mark)
- (ii) Find an equation of the normal to the curve at  $P$ . (3 marks)
- (c) (i) Find  $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7) dx$ . (3 marks)
- (ii) Hence find the equation of the curve which passes through the point  $P(1, 8)$ . (3 marks)

- 8 (a) Describe the single geometrical transformation by which the curve with equation  $y = \tan \frac{1}{2}x$  can be obtained from the curve  $y = \tan x$ . (2 marks)
- (b) Solve the equation  $\tan \frac{1}{2}x = 3$  in the interval  $0 < x < 4\pi$ , giving your answers in radians to three significant figures. (4 marks)
- (c) Solve the equation

$$\cos \theta (\sin \theta - 3 \cos \theta) = 0$$

in the interval  $0 < \theta < 2\pi$ , giving your answers in radians to three significant figures. (5 marks)

**END OF QUESTIONS**

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